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Research paper

Mathematical Modeling of Torsional and Longitudinal Oscillations in a Mine Winding Plant as a Multimass System

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Abstract

The purpose of the study is to determine the dynamic loads in the mechanical system of a mine winding plant. A 6-mass equivalent dynamic design model is constructed by reducing the number of discrete masses in the original 12-mass system by a special method. Spectra of natural oscillation frequencies of the initial and design systems are obtained. Differential equations that describe oscillations in the shaft line and ropes are drafted. Oscillation curves of the moments in the elastic coupling of the design system are obtained using numerical integration of the equations. The dynamic response factors are calculated. It was confirmed that the conversion of a 12-mass system into a 6-mass system almost does not affect the accuracy of determining the dynamic loads in the transmission line of the hoist in a given frequency range. The proposed methodology for constructing a mathematical model can be used not only for mine hoists, but also for other machines with multiple moving masses.

Key words: *Dynamics; Mathematical model; Mine winding plant; Multimass system.*

1. Introduction

Loads in the shafting and ropes under various operating modes, including during the dynamic transient processes should be determined for rational designing of a mine winding plant (MWP). For this purpose, mathematical models of the MWP are created in accordance with equivalent dynamic designs (discrete masses systems connected by elastic couplings). The mechanical transmission of the MWP contains multiple masses that rotate or progressively move. Therefore, the question what masses should be taken as discrete ones and which ones should be taken as distributed ones, reduced to discrete ones, should be solved.

2. Literature Data Analysis

In the mine winding plants, the period of unsteady motion constitutes an essential part of the full cycle of work. In this case, the dynamic loads significantly exceed the static loads. Multiple works have been devoted to solving problems related to the dynamics of transient processes in heavy machines. These studies are based on the theory of oscillations as an independent branch of science. In the last century, the complexity of mathematical calculations required significant simplifications, and multimass dynamic systems were reduced to two or three-mass. In this case, the researchers could not obtain a valid representation of the load distribution between the elements of the machine. Computers have provided ample opportunities in this field. Today, the theoretical foundations of the mechanics of the interaction of elastic linkages of mine winding plant, which are operated in the complicated difficult mining engineering conditions [1], have been created. Methods for monitoring the technical condition of hoisting equipment and improving its operational safety in the current conditions have been developed [2]. Methods have been developed for determining the estimated loads during the rope spooling and changing of the stress-strain state of hoist drum under the action of rope coils [3]. A 3-mass dynamic design of a hoisting plant with a mathematical description that takes into account the energy dissipation is developed [4]. The design dynamic models are constructed with and without regard to the elasticity of the rope. Power transition processes in the ropes of a single-rope and double-rope hoisting plants are described by an analytical differential equation [5]. Dynamic parameters of the mine hoists which determine the emergency risk are established. This includes taking into account the technical state of the individual linkages of the plants [6]. Numerical modeling allows investigating negative resonant phenomena in the mechanical system of the MWP. Recommendations have been developed on how to avoid the resonance [7]. A diagnostic method for detecting detrimental vibrations in the MWP with an integrated application of amplitude, time and frequency analysis has been developed [8]. Experiments on real facilities allow analyzing the dynamic loads and tensions in the structural elements of the MWP caused by the roughness of the guide string [9]. A promising method of upgrading the MWP is the introduction of a frequency-controlled system to control the rate of lifting. For this purpose, programmable logic controllers [10] are used.
Systematization of the results of the above studies suggests that mathematical models are widely used to determine the dynamic loads in the MWP. These models are built using the known and acknowledged methods that have become classic. In particular, such methods are described in [11]. Obviously, the adequacy of the results obtained on the models, first of all, depends on how the calculated scheme corresponds to the actual machine. To do this, methods for constructing the equivalent dynamic models of the specific types of machines should be created.

3. Purpose and Objectives of the Study

The purpose of the study is to determine the dynamic loads in the mechanical system of the MWP. To achieve this goal, the following tasks should be solved: creating an initial equivalent dynamic model of a MWP with the maximum possible number of discrete masses; determining the range of natural oscillation frequencies of the original system; obtaining the calculated equivalent dynamic model from the original one by reducing the number of discrete masses; determining the frequency range of the natural oscillations of the design system; comparing it with that of the original system, and drawing conclusions regarding the justification for reducing the number of discrete masses; drawing up a system of differential equations which describe the oscillations; determining the moments in the elastic couplings under oscillations and comparing them with static moments.

4. Mathematical Modeling of Oscillations in a Mine Winding Plant

In the article, an analytical method is used to solve systems of differential equations, which allows obtaining the characteristic equation. A numerical method is used to find the frequency spectrum of the natural oscillations of the system by solving the characteristic equation. This method is also used to integrate differential equations. As a result, the moment oscillations curves in the elastic couplings of MWP are obtained.

4.1. Original Equivalent Dynamic Model

A MWP with a discontinuous cylindrical drum was considered which has the following main characteristics: lifting height: 676 m, drum diameter: 6.3 m; width of the fixed part of the drum: 3.15 m; width of the movable part of the drum: 1.25 m; diameter of the rope: 60.5 mm, weight of the conveyance (cage): 8,130 kg; carrying capacity of the cage: 13,200 kg. In accordance with the kinematic diagram shown in Fig. 1, an original 12-mass equivalent dynamical diagram of the MWP was drawn (Fig. 2). Longitudinal oscillations of the ropes are reduced to torsion. Moments of inertia and stiffness of the sections are scaled to the drum shaft which is accepted as the main site. The oscillations are described by a system of ordinary heterogeneous second-order differential equations with constant coefficients. The system is composed using the Lagrange equations of the second kind. After an analytical solution of the differential equations and the series of transformations, the characteristic equation was obtained in the form of a determinant for calculating the natural oscillation frequencies of the system (1). The numerical values of I_i (kg·m^2) and $C_{i,j}$ (N·m) were obtained from the technical characteristics of the components and the results of calculations, based on the geometric dimensions of the units.

4.2. Mathematical Dynamic Model of a Mine Winding Plant

To reduce the number of discrete masses in the original system (Figure 2), a method based on the recommendations was applied [11]. The characteristic links are distinguished: type A and type B. Then one link type is replaced by another. An example is shown in Figure 3. The new link should have the same influence on the entire mechanical system as the one being replaced. For this purpose, the following condition is met:

$$\frac{\beta^2_{A(B)}}{\beta^2_{lim}} \geq 10 \qquad (2)$$

where $\beta_{A(B)}$ is the natural oscillation frequency of the replaced link, β_{lim} is the highest limiting frequency of free oscillations of the original equivalent model which may not be distorted. In our case, it is adopted $\beta_{lim} = \beta_7 = 132.78$ rad/s.

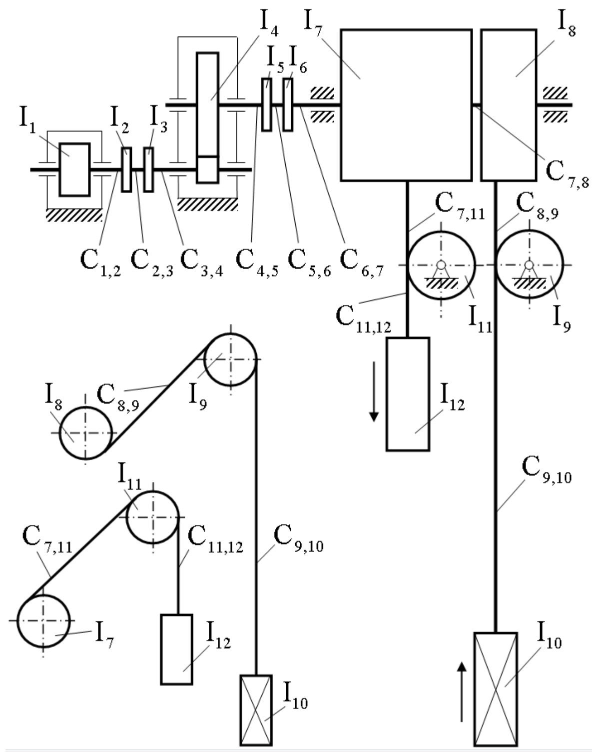

Fig. 1: Kinematic diagram of the MWP and the location of the ropes: Moments of inertia of discrete masses: I_1 – the rotor of the electric motor, I_2, I_3 – the half-couplings of the spring coupler, I_4 – single-stage cylindrical reducer, I_5, I_6 – half-couplings of the gear coupling, I_7 – fixed drum, I_8 – movable drum, I_9, I_{11} – hoisting pulleys, I_{10}, I_{12} – conveyances. $C_{i,j}$ are the coefficients of the torsional stiffness of the elastic connections between the relevant discrete masses.

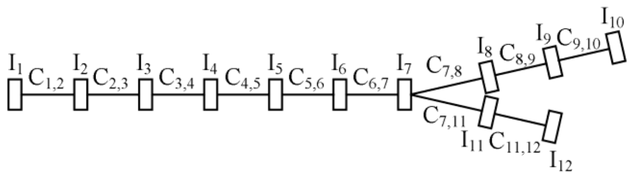

Fig. 2: The original 12-mass equivalent dynamic diagram of the MWP. The designations correspond to Figure 1.

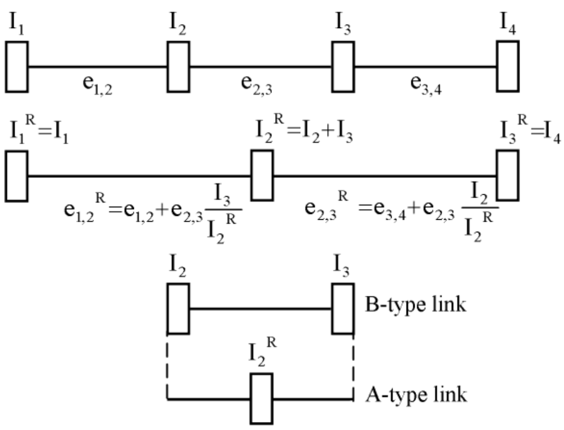

Fig. 3: Example of reducing the number of discrete masses by replacing a type B link by a type A link: I_i, I_i^R – moments of inertia before and after mass reduction, $e_{i,j}$, $e_{i,j}^R$ – compliance coefficients of elastic couplings before and after mass reduction ($e_{i,j} = 1/C_{i,j}$, $e_{i,j}^R = 1/C_{i,j}^R$).

$$\begin{vmatrix}
B_{1,2} & -\frac{C_{1,2}}{I_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{C_{2,3}}{I_2} & B_{2,3} & -\frac{C_{2,3}}{I_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{C_{3,4}}{I_3} & B_{3,4} & -\frac{C_{3,4}}{I_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{C_{4,5}}{I_4} & B_{4,5} & -\frac{C_{4,5}}{I_5} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{C_{5,6}}{I_5} & B_{5,6} & -\frac{C_{5,6}}{I_6} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{C_{6,7}}{I_6} & B_{6,7} & -\frac{C_{6,7}}{I_7} & 0 & 0 & -\frac{C_{6,7}}{I_7} & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{C_{7,8}}{I_7} & B_{7,8} & -\frac{C_{7,8}}{I_8} & 0 & \frac{C_{7,8}}{I_7} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{C_{8,9}}{I_8} & B_{8,9} & -\frac{C_{8,9}}{I_9} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{C_{9,10}}{I_9} & B_{9,10} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{C_{7,11}}{I_7} & \frac{C_{7,11}}{I_7} & 0 & 0 & B_{7,11} & -\frac{C_{7,11}}{I_{11}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{C_{11,12}}{I_{11}} & B_{11,12}
\end{vmatrix} = 0, \qquad (1)$$

where the designations mean as follows: $B_{1,2}=C_{1,2}(1/I_1+1/I_2)-\beta^2$, $B_{2,3}=C_{2,3}(1/I_2+1/I_3)-\beta^2$, $B_{3,4}=C_{3,4}(1/I_3+1/I_4)-\beta^2$, $B_{4,5}=C_{4,5}(1/I_4+1/I_5)-\beta^2$, $B_{5,6}=C_{5,6}(1/I_5+1/I_6)-\beta^2$, $B_{6,7}=C_{6,7}(1/I_6+1/I_7)-\beta^2$, $B_{7,8}=C_{7,8}(1/I_7+1/I_8)-\beta^2$, $B_{8,9}=C_{8,9}(1/I_8+1/I_9)-\beta^2$, $B_{9,10}=C_{9,10}(1/I_9+1/I_{10})-\beta^2$, $B_{7,11}=C_{7,11}(1/I_7+1/I_{11})-\beta^2$, $B_{11,12}=C_{11,12}(1/I_{11}+1/I_{12})-\beta^2$,
β - frequency of the own torsional oscillations of the system measured (rad/s). The remaining designations correspond to Figure 1.

The original 12-mass system was reduced to the calculated 6-mass, which is shown in Figure 4.

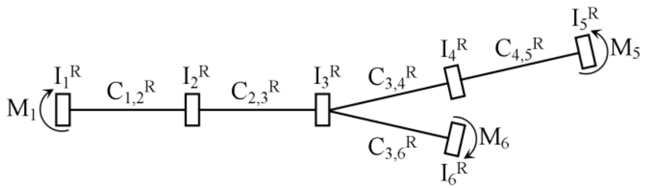

Fig. 4: Calculated 6-mass equivalent dynamic model of the MWP: Moments of inertia of discrete masses: I_1^R – rotor of the electric motor, I_2^R – reducer; I_3^R – fixed drum, I_4^R – movable drum; I_5^R, I_6^R – conveyances. $C_{i,j}^R$ are the coefficients of the torsional stiffness of the elastic connections between the relevant discrete masses, M_1, M_5, M_6 are external moments that are applied to the motor rotor and conveyances.

The oscillations of the resulting system are described by a system of differential equations (3), and the characteristic equation (4) is derived for calculating the natural oscillation frequencies of the system.

$$\left.\begin{aligned}
\ddot{M}_{1,2} &= -C_{1,2}^R\left(\frac{1}{I_1^R}+\frac{1}{I_2^R}\right)M_{1,2}+\frac{C_{1,2}^R}{I_2^R}M_{2,3}+\frac{C_{1,2}^R}{I_1^R}M_1 \\
\ddot{M}_{2,3} &= \frac{C_{2,3}^R}{I_2^R}M_{1,2}-C_{2,3}^R\left(\frac{1}{I_2^R}+\frac{1}{I_3^R}\right)M_{2,3}+\frac{C_{2,3}^R}{I_3^R}M_{3,4}+\frac{C_{2,3}^R}{I_3^R}M_{3,6} \\
\ddot{M}_{3,4} &= \frac{C_{3,4}^R}{I_3^R}M_{2,3}-C_{3,4}^R\left(\frac{1}{I_3^R}+\frac{1}{I_4^R}\right)M_{3,4}+\frac{C_{3,4}^R}{I_4^R}M_{4,5}-\frac{C_{3,4}^R}{I_3}M_{3,6} \\
\ddot{M}_{4,5} &= \frac{C_{4,5}^R}{I_4^R}M_{3,4}-C_{4,5}^R\left(\frac{1}{I_4^R}+\frac{1}{I_5^R}\right)M_{4,5}-\frac{C_{4,5}^R}{I_5^R}M_5 \\
\ddot{M}_{3,6} &= \frac{C_{3,6}^R}{I_3^R}M_{2,3}-\frac{C_{3,6}^R}{I_3^R}M_{3,4}-C_{3,6}^R\left(\frac{1}{I_3^R}+\frac{1}{I_6^R}\right)M_{3,6}-\frac{C_{3,6}^R}{I_6^R}M_6
\end{aligned}\right\} \qquad (3)$$

where $M_{1,2}$, $M_{2,3}$, …, $M_{3,6}$ are moments in the elastic couplings between the corresponding discrete masses. The remaining designations correspond to Fig. 4.

$$\begin{vmatrix}
B_{1,2}^R & -\frac{C_{1,2}^R}{I_2^R} & 0 & 0 & 0 \\
-\frac{C_{2,3}^R}{I_2^R} & B_{2,3}^R & -\frac{C_{2,3}^R}{I_3^R} & 0 & -\frac{C_{2,3}^R}{I_3^r} \\
0 & -\frac{C_{3,4}^R}{I_3^R} & B_{3,4}^R & -\frac{C_{3,4}^R}{I_4^R} & \frac{C_{3,4}^R}{I_3^R} \\
0 & 0 & -\frac{C_{4,5}^R}{I_4^R} & B_{4,5}^R & 0 \\
0 & -\frac{C_{3,6}^R}{I_3^R} & -\frac{C_{3,6}^R}{I_3^R} & 0 & B_{3,6}^R
\end{vmatrix} = 0, \qquad (4)$$

where are the following designations:
$B_{1,2}^R=C_{1,2}^R(1/I_1^R+1/I_2^R)-\beta^2$, $B_{2,3}^R=C_{2,3}^R(1/I_2^R+1/I_3^R)-\beta^2$,
$B_{3,4}^R=C_{3,4}^R(1/I_3^R+1/I_4^R)-\beta^2$, $B_{4,5}^R=C_{4,5}^R(1/I_4^R+1/I_5^R)-\beta^2$,
$B_{3,6}^R=C_{3,6}^R(1/I_3^R+1/I_6^R)-\beta^2$,
β is the natural frequency of the system.
The remaining designations correspond to Figure 4.

5. The Results of Research on the Mathematical Model

After solving the equation (1), the frequency range of the natural oscillation frequencies of the original 12-mass system, which is depicted in Figure 2, was found by a numerical method. These are the frequencies in rad/s:
$\beta_1 = 3.11$, $\beta_2 = 13.06$, $\beta_3 = 23.83$, $\beta_4 = 29.90$,
$\beta_5 = 54.15$, $\beta_6 = 115.98$, $\beta_7 = 132.78$, $\beta_8 = 483.82$,
$\beta_9 = 925.88$, $\beta_{10} = 966.64$, $\beta_{11} = 2053.53$.

By solving equation (4), the frequency spectrum of the natural oscillations of the calculated 6-mass system (Figure 4) in rad/s is obtained: $\beta_1^R = 3.07$, $\beta_2^R = 12.68$, $\beta_3^R = 22.61$, $\beta_4^R = 115.06$; $\beta_5^R = 124.61$.
Figure 5 shows the oscillation moment curves in the elastic couplings of the MWP obtained by numerical integration of the equations system (3). The oscillations are caused by the instantaneous application of constant external moments to the motor rotor (M_1) and conveyances (M_5, M_6) at the beginning of lifting with an acceleration of 0.75 m/s^2. The dynamic coefficients will be determined by the formula (5).

$$K_{i,j} = \frac{M_{i,j}}{M_{i,j}^S} \qquad (5)$$

where
$M_{i,j}$ is the maximum moment in the elastic connection under oscillations (Figure 5),
Mi,jS is the moment in the same elastic connection in the static state.
Calculation by formula (5) gives the following results:

$K_{1,2} = 2.73$, $K_{2,3} = 2.82$, $K_{3,4} = 2.32$, $K_{4,5} = 2.12$, $K_{3,6} = 1.71$.

6. Discussion of the Results of the Studies of the Dynamics of a Mine Winding Plant Using a Mathematical Model

According the results obtained by solving the characteristic equations (1) and (4), the proposed procedure for reducing the number of discrete masses allows constructing an equivalent model that underlies the mathematical dynamic model of a MWP. This corresponds to the purpose of the study. The curves reflect the loads in the mechanical system during the transient processes at the beginning of the ascent. The coefficients calculated from formula (5) allow us to estimate how much the dynamic loads differ from the static state. This should be taken into account when designing the plant. The results are consistent with the studies ofother authors, in particular [1, 2, 4, 5]. It should be noted that when creating a mathematical model, traditional assumptions are adopted that are not beyond the limits of the required accuracy. For example: ropes are considered as flexible weightless elastic couplings, the mass of 1/3 of the length of each rope is scaled to the corresponding end load and drum, the stiffness of the elastic couplings are constant in time, the dissipative forces are not taken into account, since they have slight effect in the first period of oscillations.

7. Conclusion

In this article, a method is proposed for obtaining an equivalent dynamic model of a mine winding plant, which enables reduction of the number of discrete masses from 12 to 6. The range of own frequencies of torsional oscillations by a 6-mass sufficiently degree of accuracy coincides with the lower frequencies of the original system. Therefore we may assert that the conversion of a 12-mass system into a 6-mass system has virtually no effect on the accuracy of determining dynamic loads in the transmission line of the hoist in a given frequency range. The dynamic coefficients in elastic couplings under the oscillations caused by the instantaneous application of constant external moments to the motor rotor and the end loads at the beginning of the lift are 2.32 to 2.82 in the shaft line and 1.71 to 2.12 in the ropes. This should be taken into account when designing mine winding plants. The proposed methodology for constructing a mathematical model can be used not only for mine hoists, but also for other machines with multiple moving masses.

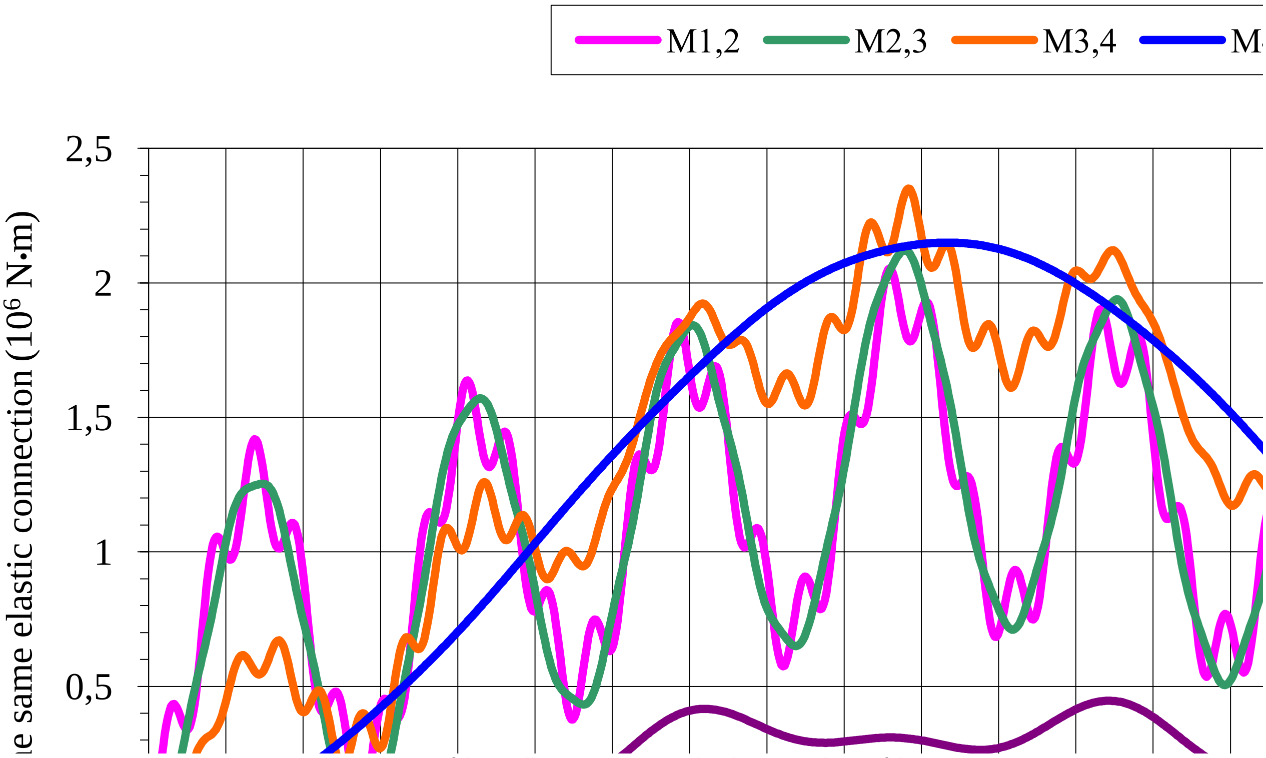

Fig. 5: The diagram of the oscillation moments in the elastic couplings of the MWP.

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