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Features of creating systems development for obtaining mathematical models of heat exchangers

The paper presents analytical mathematical models of heat exchangers, based on typical hydrodynamic models of ideal mixing and ideal exclusion. These models allow researching the influence of technological parameters on the heat exchange process and determining the dynamic properties of heat exchangers through various channels. Mathematical models are implemented in the MATLAB environment and can be used in industrial control system (ICS) and computer-aided design (CAD) systems.
Keywords*: heat exchanger, coolant, ideal mixing, ideal exclusion, MATLAB, ICS, CAD.*

Introduction

Heat-exchange apparatus is a very significant part of technological equipment in many branches of technology: in power engineering, chemical, metallurgical, petroleum, pharmaceutical, food and other industries. The share of heat exchange equipment in the chemical industry averages 15-18%, in the petrochemical and oil refining industry – 50% [1].

The most widely used among the variety of existing devices are recuperative heat exchangers, in which heat from one coolant is transferred to another through the wall that separates them. These include heat exchangers made from pipes of various shapes, lamellar apparatus and apparatus, which are parts of chemical reactors, distillation columns, etc.

The real structure of the flow of refrigerants in the recuperative apparatuses is very complex, which complicates the construction of their analytical mathematical models. The heat exchanger mathematical model can be significantly simplified if assume that the structure of coolant flows in it corresponds to typical hydrodynamic models, the simplest of which are models of ideal mixing and ideal exclusion [2].

Main part

The heat transfer process from a hot coolant to a cold one through a wall that separates them consists of a series of steps:

– transfer of heat from the hot coolant to a colder wall,
– absorption of heat by the wall and its heating,
– transfer of heat from the heated wall to the cold coolant.

The intensity of heat transfer from one coolant to another depends on the temperature difference between them and on the thermal resistance.

The total heat flow q is determined by the following relationship

$$q = \alpha F \Delta T,$$

where α is a heat transfer coefficient, (W/m^{2}K);
F is a heat transfer surface, m^2;
ΔT is a temperature difference between coolants, K.

In practice, the intensity of heat transfer between fluids in recuperative heat exchangers is calculated using the heat transfer coefficient K_T:

$$q = K_T F \Delta T,$$

$$K_T = \frac{1}{\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{\delta_{wl}}{\lambda_{wl}} + R},$$

where α_1 and α_2 are heat transfer coefficients, W/(m^{2}K);

λ_{wl} is a thermal conduction of wall material, W/(mK);

δ_{wl} is a wall thickness, m;

R is the thermal resistance of pollution, (m^{2}K)/W.

The mathematical description of the process in heat exchangers is conveniently written as an expression that characterizes the temperature change in the coolant flow over time. This is due, firstly, to the movement of the flow and, secondly, to heat transfer [3].

If we assume that the structure of the coolant flow corresponds to the model of ideal mixing, then as the basis of the mathematical model can be chosen the hydrodynamic model of the following type:

$$Vc_T\rho\frac{dT}{dt}=\upsilon c_T\rho(T_{\text{VX}}-T), \qquad (1)$$

where c_T is a specific heat capacity of coolant, J/(kgK);

ρ is a coolant density, kg/m^3;

V is a volume of ideal mixing zone, m^3;

υ is a volumetric flow rate, m^3/s;

T_{VX} is inlet temperature to the ideal mixing zone, K;

T is the temperature at any point in the ideal mixing zone and at the exit, K;

t is time, s.

Equation (1) characterizes temperature change in the flow due to the coolant movement.

To take into account the temperature change due to heat transfer, it is necessary add to equation the heat transfer intensity in reaction volume Vq_T.

$$Vc_T\rho\frac{dT}{dt}=\upsilon c_T\rho(T_{\text{VX}}-T)+Vq_T.$$

For a recuperative heat exchanger of ideal mixing, the heat exchange intensity is calculated by the formula:

$$Vq_T=K_T F\Delta T.$$

If there is no mixing, and the coolant flow structure corresponds to the ideal exclusion model, then for its mathematically describe can be used a hydrodynamic model of ideal exclusion:

$$S_B\rho c_P\frac{dT}{dt}=-u\rho c_T\frac{dT}{dl},$$

where S_B is section area of ideal exclusion zone, m^2;

u is linear flow rate, m/s;

l is length of ideal exclusion zone, m.

With given the heat transfer, we obtain the equation

$$S_B\rho c_T\frac{dT}{dt}=-u\rho c_T\frac{dT}{dl}+\frac{F}{L}K_T\Delta T.$$

The block diagram of the heat exchanger of the “mixing-mixing” type is shown in Fig. 1.

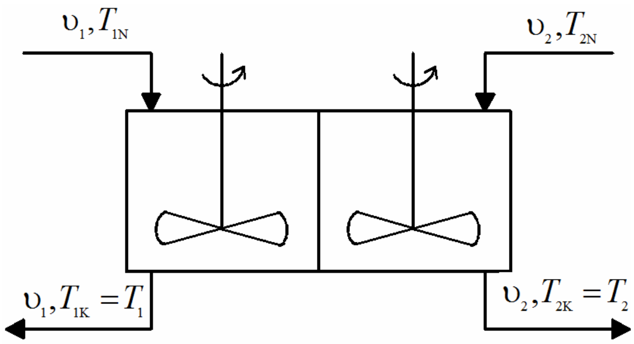

Fig. 1. Block diagram of the heat exchanger “mixing-mixing”

The mathematical model of the heat exchanger “mixing-mixing” has the following form:

$$\begin{cases} V_1c_{1T}\rho_1\dfrac{dT_1}{dt}=\upsilon_1c_{1T}\rho_1(T_{1N}-T_1)-kF(T_1-T_2), \\ V_2c_{2T}\rho_2\dfrac{dT_2}{dt}=\upsilon_2c_{1T}\rho_2(T_{2N}-T_2)+kF(T_1-T_2), \end{cases} \qquad (2)$$

The mathematical model (2) is a system of ordinary differential equations with constant coefficients. The solution of such systems can be carried out by numerical methods. To do this, need to bring system (2) to the Cauchy problem. In this case, the system takes the form

$$\begin{cases} \dfrac{dT_1}{dt}=\dfrac{\upsilon_1}{V_1}(T_{1N}-T_1)-\dfrac{kF}{V_1c_{1T}\rho_1}(T_1-T_2) \\ \dfrac{dT_2}{dt}=\dfrac{\upsilon_2}{V_2}(T_{2N}-T_2)+\dfrac{kF}{V_2c_{2T}\rho_2}(T_1-T_2) \end{cases} \qquad (3)$$

The initial conditions (coolant temperatures at heat exchanger exit in steady state), which are necessary for the solution, are find from the mathematical model of the stationary mode of the heat exchanger:

$$\begin{cases} \upsilon_1c_{1T}\rho_1(T_{1N}-T_1)-kF(T_1-T_2)=0 \\ \upsilon_2c_{2T}\rho_2(T_{2N}-T_2)+kF(T_1-T_2)=0 \end{cases} \qquad (4)$$

Solving system (4) with respect to the unknowns T_1 and T_2, determine the initial conditions T_1^0 and T_2^0.

For the numerical solution of the system of differential equations (3) in the MATLAB environment, the built-in function ode45 is used, which implements a fifth-order Runge-Kutta method with a variable step. The function has the following syntax:

```
[time,T]=ode45(@Tepl_PP, [0 tk], [T1_0 T2_0]),
```

where [time, T] is the solution matrix (time dependence of the temperature at heat exchanger outlet);

@Tepl_PP is the subfunction that containing the description of the right-hand sides of the differential equations of the system (3),

tk is the integration interval;

T1_0 and T2_0 is initial conditions.

The task of modeling the heat exchanger is finding the coolant temperature at the exit. As a result of computer simulation, it is possible to obtain step response of a heat exchanger by control channels. Fig. 2 shows the results of the calculation of the heat exchanger for the following initial data: T_{1N} =115, T_{2N} =10, υ_1= 4,94e-3, υ_2 = 5,43e-3, c_{1T} = 3,75e3, c_{2T} =3,14e3, ρ_1=850 , ρ_2 = 920 , k =4360 , F =4, $V_1 = V_2$ =2,5.

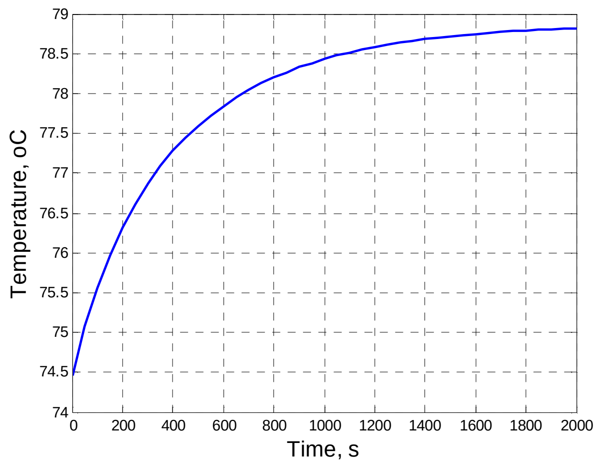

Fig. 2. Step response of the heat exchanger by the channel “inlet hot coolant flow rate – outlet coolant temperature” at 20% input disturbance.

Heat exchangers of the “pipe in pipe” type (Fig. 3) are described by hydrodynamic models of ideal exclusion [4]. Figure 3 shows the cocurrent heat exchanger.

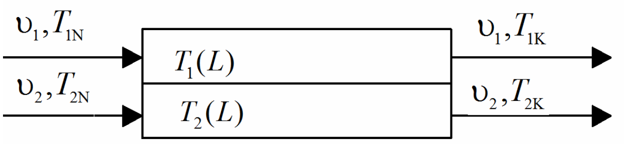

Fig. 3. Diagram of a cocurrent heat exchanger “pipe in pipe”

The mathematical model of the heat exchanger dynamics of the “exclusion-exclusion” type is a system of two partial differential equations and is a model with distributed parameters.

$$\begin{cases} S_{1B}c_{1T}\dfrac{\partial T_1}{\partial t} = -\upsilon_1 c_{1T}\dfrac{\partial T_1}{\partial l} - \dfrac{F}{L}K_T\Delta T; \\ S_{2B}c_{2T}\dfrac{\partial T_2}{\partial t} = -\upsilon_2 c_{2T}\dfrac{\partial T_2}{\partial l} + \dfrac{F}{L}K_T\Delta T, \end{cases}$$

where $\Delta T = T_1 - T_2$, T_1, T_2 change along the length of the corresponding zone of ideal extrusion.

In the stationary mode of heat transfer $\dfrac{\partial T}{\partial t} = 0$, a mathematical model of statics is obtained.

$$\begin{cases} \upsilon_1\rho_1 c_{T1}\dfrac{\partial T_1}{\partial l} = -\dfrac{F}{L}K_T(T_1 - T_2) \\ \upsilon_2\rho_2 c_{T2}\dfrac{\partial T_2}{\partial l} = \dfrac{F}{L}K_T(T_1 - T_2) \end{cases}$$

Given that the pipe surface is $F = \pi DL$, we obtain the system of equations:

$$\begin{cases} \dfrac{\partial T_1}{\partial l} = -\dfrac{K_T\pi D_1}{\upsilon_1\rho_1 c_{T1}}(T_1 - T_2) \\ \dfrac{\partial T_2}{\partial l} = \dfrac{K_T\pi D_1}{\upsilon_2\rho_2 c_{T2}}(T_1 - T_2) \end{cases} \qquad (5)$$

The mathematical model of statics (5) allows calculating the heat exchanger length, necessary for effective cooling and drawing graphs of temperature changes of coolants along the length of the pipes.

The system of equations (5) corresponds to the statement of the Cauchy problem. The initial conditions for the calculation are the coolant temperature at heat exchanger inlet.

The ode45 function for our case is as follows:

```
[length,T]=ode45(@func_Tr1, [0 L], [T10 T20]),
```

where [length, T] is the solution matrix (temperature dependence of the heat exchanger length);

@ func_Tr is function of the right parts of the system (5),

[0 L] is the integration interval;

[T10 T20] is vector of initial conditions.

The results of the computer simulation carried out in the MATLAB environment for the following initial data: T_1^0 =170, T_2^0 =15, υ_1 = 2.28e-5, υ_2 = 2.75e-5, c_{1T} = 3350, c_{2T} =4190, ρ_1 =890 , ρ_2 =910 , k =4190 , D_1 ==0.01, are shown in Fig. 4.

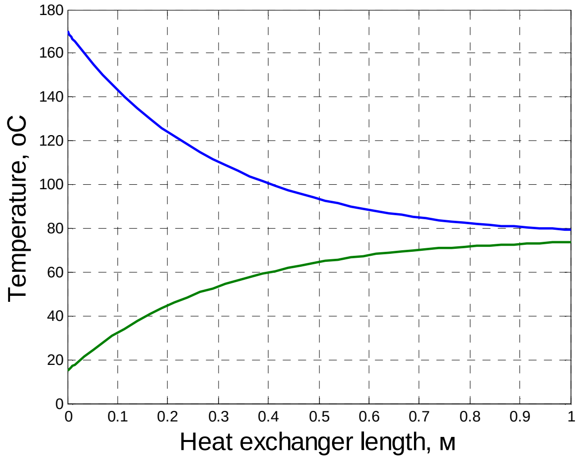

Fig. 4. Temperature variation along the length of the "pipe in pipe" heat exchanger

Conclusions

As a result of the research, a methods for constructing analytical mathematical modeling of heat exchangers, that based on idealized hydrodynamic models of the motion of coolant is proposed.

The obtained mathematical models can be used in CAD systems to identify the dynamic properties of heat exchangers in automated process control systems, and to study the effect of temperature, flow rates of coolant and refrigerant, plant dimension on the heat exchange process.

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Красников І. Л., Герман Э. Е., Бабиченко Ю. К. Особенности создания автоматизированных систем получения математических моделей теплообменных аппаратов.

Аннотация. Представлены аналитические математические модели теплообменных аппаратов, основанные на типовых гидродинамических моделях идеального перемешивания и идеального вытеснения. Модели позволяют исследовать влияние технологических параметров на процесс теплообмена и определять динамические свойства теплообменников по различным каналам. Математические модели реализованы в среде MATLAB и могут быть использованы в автоматизированных системах управления технологическими процессами (АСУТП) и в системах автоматизированного проектирования (САПР).

Ключевые слова: теплообменник, теплоноситель, модель идеального перемешивания, модель идеального вытеснения, MATLAB, АСУТП, САПР.

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Красніков І. Л., Герман Е. Є., Бабіченко Ю. А. Особливості створення автоматизованих систем отримання математичних моделей теплообмінних апаратів.

Анотація. Представлені аналітичні математичні моделі теплообмінних апаратів, що засновані на типових гідродинамічних моделях ідеального змішування та ідеального витіснення. Ці моделі дозволяють досліджувати вплив технологічних параметрів на процес теплообміну і визначати динамічні властивості теплообмінних апаратів по різних каналах. Математичні моделі реалізовані в середовищі MATLAB і можуть бути використані в автоматизованих системах управління технологічними процесами (АСУТП) та в системах автоматизованого проектування (САПР).

Ключові слова: теплообмінник, теплоносій, модель ідеального змішування, модель ідеального витіснення, MATLAB, АСУТП, САПР.

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