# PROPERTIES OF HIGH–SPIN BOSON INTERACTION CURRENTS AND ELIMINATION OF POWER DIVERGENCES


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The problem of the elimination of the power divergences for the interactions of the high-spin bosons ( $J \geq 1$ ) is investigated. It is proved that in the consistent theory the high-spin boson interaction currents and the field tensors must obey similar requirements. Therefore the momentum dependencies of the propagators for all the bosons are the same. The partial differential equations derived for some components include the derivatives of order $2J$ for the currents. Therefore the current components for the spin- $J$ boson must decrease with the momentum $|p_\nu| \rightarrow \infty$ at least as $|p_\nu|^{-2J}$

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## INTRODUCTION

The quantum electrodynamics describes very well the interactions of the photons (massless vector particles) and the electrons (spin-1/2 particles). The electromagnetic interactions of the spinless massive particles are investigated in [1].

Such interaction was used in the generalizations of the quantum chromodynamics and in the theory of the electroweak interactions (see for example [2]). These theories are based on the interactions of the gauge massless vector bosons, the spinless boson and the spin-$\frac{1}{2}$ particles. The vector $W^{\pm}$ - and $Z^0$ - bosons became massive by means of the Higgs effect. However, the observed massive vector particles $\rho, \omega^0, \varphi^0, K^*, a_1(1260)$, the tensor particles $f_2(1270), a_2(1320), f_2'(1525), k^*(1430)$, as well as the $P_{33}(1232)$ and other nucleon resonances are not described by the gauge fields. In the conventional models of the massive high-spin hadrons (J≥1) the interaction vertex functions are written with the coupling constants at the requirement of the relativistic invariance only. Such models result in the power divergences for the real and virtual high-spin hadrons. In particular the conventional models give the cross-sections of the processes involving the massive high-spin hadrons increasing with the energy growth. It contradicts to the experimental data. For example, if in some reaction of the pion-nucleon interaction to change the pion by high-spin boson or the nucleon by the high-spin nucleon resonance ($P_{33}$(1232), $D_{13}$(1520) and other) then the cross-sections of new reactions in the experiment decrease rapidly with energy. Besides, such models give ambiguous expressions for amplitudes, as shown in [3]. Although the high-spin particles are investigated in quite a number of approaches (e g., refs [ 4-7]), the problem of the elimination of power divergences for them interactions is .not solved.

In this paper we prove the theorems which can permit one to eliminate the power divergences for the contributions of the high–spin bosons (HSB) to the amplitudes.

Let $U(x)_{\mu_1 \ldots \mu_l} = U(x)_{\{\mu\}}$ is the symmetric tensor for spin $J = l$ boson field. This tensor must satisfy the conditions

$$\partial_{\mu_k} U(x)_{\mu_1 \ldots \mu_l} = 0 \qquad (1.a)$$

$$g_{\mu_j \mu_k} U(x)_{\mu_1 \ldots \mu_l} = 0 \qquad (1.b)$$

where *j,k=1,2,…,l*. The equations for the field $U(x)^l_{\{\mu\}}$ are given by

$$( \ +M^2) U(x)_{\mu_1 \ldots \mu_l} = j(x)_{\mu_1 \ldots \mu_l} \qquad (2)$$

where $j(x)_{\mu_1 \ldots \mu_l}$ is the tensor of the currents for HSB interactions, and M is the HSB mass. The properties of these currents determine the convergence or the divergence of the integrals in the amplitudes for the HSB contributions and the energy decrease or increasing in the reactions involving HSB.

In the conventional models there are two sources of power divergences. The first of them is related to the $p_\mu p_\nu / M^2$ -type factors in the numerators of the HSB propagators. The second source is due to the fact that the vertex functions of the HSB interactions include the HSB momentum $p_\mu$ . The amplitudes corresponding to the contributions of the virtual HSB.

## 1. THEOREM ON CURRENTS AND FIELDS

If the components of the field tensor $U(x)^{l}_{\{\mu\}}$ are continuous and have the continuous partial derivatives up to third order as well as the components of the current tensor $j(x)^{l}_{\{\mu\}}$ are continuous and have the continuous partial derivatives then the current tensor satisfy the conditions

$$\partial_{\mu_k} j(x)_{\mu_1 \ldots \mu_l} = 0 \tag{3.a}$$

$$g_{\mu_j \mu_k} j(x)_{\mu_1 \ldots \mu_l} = 0 \tag{3.b}$$

To prove the condition (*3.a*) we find the partial derivatives of the eq.(2) and sum with respect to $\mu_k$. In consequence of the condition (*1.a*) we derive (*3.a*).

Now to prove Eq. (3.b) we convolve the Eq. (2) with respect to the $\mu_j$, $\mu_k$ indices and use the condition Eq. (1.b).

In the momentum representation the conditions (1) and (3) are written as

$$p_{\mu_k} U(p)_{\mu_1 \ldots \mu_l} = 0 \tag{4.a}$$

$$g_{\mu_j \mu_k} U(p)_{\mu_1 \ldots \mu_l} = 0 \tag{4.b}$$

$$p_{\mu_k} j(p)_{\mu_1 \ldots \mu_l} = 0 \tag{5.a}$$

$$g_{\mu_j \mu_k} j(p)_{\mu_1 \ldots \mu_l} = 0 \tag{5.b}$$

where p is the 4-momentum of HSB, $U(p)_{\mu_1 \ldots \mu_l} = 0$ and $j(p)_{\mu_1 \ldots \mu_l} = 0$ are the Fourier components of the field and current tensors, respectively. Note that the condition (*5.a*) was proposed in [3,8].

It is clear that the models including the conditions (1) and (4) but without the conditions (3) and (5) are contradictory. The consistent theory of the HSB interactions must be based on the conditions (1) and (3) ((4) and (5)).

To derive the current tensors $j(p)_{\mu_1 \ldots \mu_l}$, which obey the conditions (5) we use the contracted projection operator $\Pi^{l}(p,a,b)$ [9] for arbitrary 4-vectors *a* and *b*:

$$\Pi^{l}(p,a,b) = a_{\mu_1} \ldots a_{\mu_l} \Pi(p)_{\mu_1 \cdots \mu_l, \nu_1 \cdots \nu_l} b_{\nu_1} \ldots b_{\nu_l}$$
$$= \frac{l!}{(2l-1)!!} \left(-\tilde{a}^2\right)^{\frac{l}{2}} \left(-\tilde{b}^2\right)^{\frac{l}{2}} P_l(z) \tag{6}$$

where $P_l(z)$ are the Legendre polynomials and

$$\tilde{a}_\mu = a_\mu - p_\mu \frac{(pa)}{p^2}, \quad \tilde{b}_\mu = b_\mu - p_\mu \frac{(pb)}{p^2},$$
$$(\tilde{a}p) = (\tilde{b}p) = 0, \quad z = \frac{-(\tilde{a}\tilde{b})}{\sqrt{-\tilde{a}^2}\sqrt{-\tilde{b}^2}}. \tag{7}$$

The projection operator $\Pi(p)_{\mu_1 \cdots \mu_l, \nu_1 \cdots \nu_l}$ can be obtained from $\Pi^{l}(p,a,b)$ by means of the partial derivatives with respect to $a_{\mu_k}$ and $b_{\nu_j}$.

The current tensors $j(p)_{\mu_1 \cdots \mu_l}$, which satisfy (5), may be written as

$$j(p)_{\mu_1 \cdots \mu_l} = \Pi(p)_{\mu_1 \cdots \mu_l, \nu_1 \cdots \nu_l} \eta(p)_{\nu_1 \cdots \nu_l} \tag{8}$$

where $\eta(p)_{\nu_1 \cdots \nu_l}$ is an arbitrary current tensor.

## 2. THEOREM ON ASYMPTOTE OF CURRENTS

If for the HSB interactions the Eq. (2), the conditions (1),(3) are valid as well as the components of the field tensor $U(x)_{\mu_1 \ldots \mu_l}$ and the current tensor $j(x)_{\mu_1 \ldots \mu_l}$ are continuous and have the continuos partial derivatives up to the orders $2l+2$ and $2l$, respectively, then the components of the current tensor $j(p)_{\mu_1 \ldots \mu_l}$ must decrease with $|p_\nu| \to \infty$ at least as $|p_\nu|^{-2l}$.

Indeed, the equations (2) with the conditions (1) and (3) form the overdetermined system of the linear partial differential and algebraic equations. Such systems of equations can be reduced to the equations for one component, or some linear combinations of the components. But the order of the differential equation increases. To derive $\mathbf{2}l+\mathbf{1}$ such differential equations for the system (1), (2), (3) we multiply the eq. (2) on the projection operator $\Pi(x)_{\mu_1 \ldots \mu_l, \nu_1 \ldots \nu_l}$ and $^{l}$.

$$^{l}\Pi(x)_{\mu_1 \ldots \mu_l, \nu_1 \ldots \nu_l} ( \ +M^2) U(x)_{\mu_1 \ldots \mu_l} =$$
$$= \ ^{l}\Pi(x)_{\mu_1 \ldots \mu_l, \nu_1 \ldots \nu_l} j(x)_{\nu_1 \ldots \nu_l} \tag{9}$$

We have derived $\mathbf{2}l+\mathbf{1}$ differential equations of the order $\mathbf{2}l+\mathbf{2}$. Some equations in (9) are equations for one components, but there are the equations for determined linear combination of the components also.

Consider the Fourier transformation

$$j(x)_{\mu_1 \ldots \mu_l} = \int d^4 p e^{-ipx} j(p)_{\mu_1 \ldots \mu_l} \tag{10}$$

To derive the continuous components of the currents and their partial derivatives up to order $\mathbf{2}l$ we must demand the convergence of the integrals

$$\int d^4 p e^{-ipx} j(p)_{\mu_1 \ldots \mu_l} \cdot p_{\nu_1} p_{\nu_2} \cdots p_{\nu_{2l}} \tag{11}$$

From the uniform convergence of these integrals we conclude that at $|p_\nu| \to \infty$

$$\left| j(p)_{\mu_1 \ldots \mu_l} \right| < |p_\nu|^{-2l-4} \tag{12}$$

It was the classical consideration. In the quantum field theory the renormalization procedures exist. It permits one to derive the finite amplitudes, in spite of that the interaction currents of the spinless particles have no the momentum dependence (it follows from Lagrangian of the interaction). Therefore we conclude that at $|p_\nu| \to \infty$

$$\left| j(p)_{\mu_1 \dots \mu_l} \right| < |p_\nu|^{-2l} \qquad (13)$$

In consequence of the asymptotic conditions (13) for the current components the power divergences due to the vertex functions of the HSB interactions ought to disappear.

## 3. INTERACTIONS OF HIGH-SPIN BOSONS AND SPINLESS PARTICLES

In conventional approach the current tensor of $J(p) \to O(q_1) + O(q_2)$ - transition is given by

$$\eta(p, q_1, q_2)_{\mu_1 \dots \mu_l} = g(q_1 - q_2)_{\mu_1} \dots (q_1 - q_2)_{\mu_l} \cdot \varphi^+(q_1)\varphi^+(q_2) \qquad (14)$$

where g is the coupling constant. To satisfy (5) we introduce 4-vector

$$Q_\mu = q_{1\mu} p^2 - (q_1 p) p_\mu = - q_{2\mu} p^2 + (q_2 p) p_\mu, \ (Qp) = 0 \qquad (15)$$

The current tensor $j(p)_{\mu_1 \dots \mu_l}$ may be written as

$$j(p)_{\mu_1 \dots \mu_l} = g_l f^l(P, Q) \Pi(p)_{\mu_1 \dots \mu_l, \nu_1 \dots \nu_l} Q_{\nu_1} \dots Q_{\nu_l} \varphi^+(q_1)\varphi^+(q_2) \qquad (16)$$

where $g_l$ is the coupling constant, the function $f(P,Q)$ provides the asymptotic be behavior of the currents. For this function we propose

$$f_1(P,Q) = \left(Q^4 + \Lambda^4\right)^{-\frac{1}{2}}$$

$$f_2(P,Q) = (p^4 + \Lambda^4)^{-1}$$

or

$$f_3(Q) = \exp\left[ - \left( \frac{Q^4}{\Lambda^4} + \frac{\Lambda^4}{Q^4} \right) \right] \qquad (17)$$

where $\Lambda$ is the constant. As at $|p_\nu| \to \infty$ $Q_\mu \sim |p_\nu|^{+2}$ we have

$$Q_\mu f_1(P,Q) \sim Q_\mu f_2(p,q) \sim |p_\nu|^{-2}.$$

Thus for the spin – $l$ HSB

$$j(p)_{\mu_1 \dots \mu_l} \sim |p_\nu|^{-2l} \qquad (18)$$

In particular, the transition currents of two spinless into the spin-1,2, and 3 boson may be written as

$$J = 1 \qquad j_\mu = igf(p,q) Q_\mu \varphi(q_1)\varphi(q_2)$$

$$J = 2 \quad j_{\mu\nu} = i\frac{g}{3} f^2(p,q)\left[3Q_\mu Q_\nu + d_{\mu n} Q^2\right]$$

$$J = 3 \quad j_{\mu\nu\rho} = i\frac{g}{10} f^3(p,q)\left[5Q_\mu Q_\nu Q_\rho + Q^2\left(d_{\mu\nu}Q_\rho + d_{\mu\rho}Q_\nu + d_{\nu\rho}Q_\mu\right)\right] \qquad (19)$$

## 4. HIGH-SPIN BOSON PROPAGATORS

The covariant chronological product of the HSB fields $U(x)^l{}_{\{\mu\}}$ and $U(y)^l{}_{\{\nu\}}$ must obey conditions (4) and we can write

$$< 0|T^* U(x)_{\mu_1 \dots \mu_l} U^+(y)_{\nu_1 \dots \nu_l}|0> = \frac{i}{l!} \frac{\partial}{\partial a_{\mu_1}} \dots \frac{\partial}{\partial a_{\mu_l}} \frac{\partial}{\partial b_{\nu_1}} \dots \frac{\partial}{\partial b_{\nu_l}} \cdot \frac{1}{(2l-1)!!} \cdot \int \frac{d^4 p}{(2\pi)^4} (-\tilde{a}^2)^{\frac{l}{2}} (-\tilde{b}^2)^{\frac{l}{2}} P_l(z) \frac{e^{-i(x-y)}}{p^2 - M^2 + i\varepsilon} \qquad (20)$$

In particular, for $J = 1$ we have

$$< 0|T^* U(x)_\mu U^+(y)_\nu|0> = i\int \frac{d^4 p}{(2\pi)^4} \frac{- g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2 + i\varepsilon_1}}{p^2 - M^2 + i\varepsilon} e^{-i(x-y)} \qquad (21)$$

The HSB propagators in the momentum space include the $p_\mu \ p_\nu / p^2$ - type terms. Such terms are dangerous as they lead to the power divergences. However in our approach they do not contribute to the amplitudes. Indeed, the contributions of HSB to the amplitudes are determined by the products of the HSB propagators and the interactions currents. In consequence of the condition (5.a) the contributions of the terms including the $p_\mu \ p_\nu / p^2$ - type factors to the amplitudes disappear. It is clear for $p^2 \neq 0$. The HSB propagator, as function of $p^2$, has the removable discontinuity at $p^2 = 0$. To eliminate this discontinuity we substitute

$$\frac{p_\mu p_\nu}{p^2} \to \frac{p_\mu p_\nu}{p^2 + i\varepsilon_1} \qquad (22)$$

and put $\varepsilon_1 = 0$ after the calculations. Therefore the $p_\mu \ p_\nu / p^2$ - type factors do not contribute to the amplitudes at any $p^2$. Such a way the effective pert of the HSB propagator, which gives the non-zero contribution to the amplitudes, may be written as

$$< 0|T^* U(x)_{\mu_1 \dots \mu_l} U^+(y)_{\nu_1 \dots \nu_l}|0> = c_l \, g_{\mu_1\nu_1} g_{\mu_2\nu_2} \dots g_{\mu_l\nu_l} D(x-y), \qquad (23)$$

$$c_l = \frac{(-1)^l (2l)!}{2^l l! (2l-1)!!},$$

$$D(x-y) = i\int \frac{d^4 p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2 - M^2 + i\varepsilon}$$

We can see that the effective parts of the propagators for any integer spin have the same momentum depende.

## 5. HSB CONTRIBUTION TO SELF-ENERGY OPERATOR OF SCALAR PARTICLE

Consider the consequence of our approach for the simple example: the HSB contribution to the self-energy operator $\sum_l(q)$ of the scalar particle with the momentum *q* in the one-loop approximation.

We use the currents (16) with the momenta of the virtual scalar particle and HSB are *p-q* and *p,* respectively. We have

$$\sum{}_l(q) = ig^2 \int \frac{d^4 p}{(2\pi)^4} \cdot f^{2l}(p,q) Q_{\mu_1} \ldots q_{\mu_l}$$

$$\cdot \Pi(p)^l_{\{\mu\}\{\nu\}} \frac{\Pi(p)^l_{\{\nu\}\{\rho\}} \Pi(p)^l_{\{\rho\}\{\sigma\}} Q_{\sigma_1} \ldots Q_{\sigma_l}}{(p-q)^2 - \mu^2 + i\varepsilon} \tag{24}$$

where $\mu$ is the virtual scalar particle mass,

$$Q_\mu = (qp) p_\mu - q_\mu p^2$$

For the self-energy operator $Q_\mu = Q^l_\mu$ and $z = 1$ in (7). Using the properties of the projection operator, we derive

$$\sum{}_l(q) = -ig \frac{l!}{(2l-1)!!} \int \frac{d^4 p}{(2\pi)^4} (-Q^2)^l \frac{f^{2l}(p,q)}{[p^2 - M^2 + i\varepsilon][(p-q)^2 - \mu^2 + i\varepsilon]} \tag{25}$$

As consequence of the theorem on the asymptote of the currents

$$(-Q^2)^l f^{2l}(p,q) \le |p_\mu|^{-2l} \tag{26}$$

and the integral in (25) ought to converge. We see that the necessary condition of the convergence is carried out in (25) better with the growth of the HSB spin. However we must note that the integral in (25) is the infinite multiple integral. For such integrals the cases are possible when the necessary condition of the convergence is carried out but the multiple integrals diverge. Therefore for the convergence of the integrals it is need to find the suitable function $f(p,q)$.

## 6. CONCLUSION

We have shown that in the consistent in theory of the HSB interactions must be valid simultaneously the conditions (1) and (3) ((4) and (5)).

Condition (5.a) permits one to eliminate the factors of the $p_{\mu_j} p_{\nu_k}$ - type in the propagators of HSB and leads to the same momentum dependence of any spin boson propagators.

In consequence of the theorem on the current asymptote the divergences due to the vertex functions of the HSB current interactions can be eliminated.

We can expect now that the cross-sections of the reactions involving HSB in contrast with the predictions of the conventional approach, will decrease with the energy growth. Moreover, this decreasing will intensify with the growth of the boson spin.

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